E 6053 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2018

First Semester

Complementary Course—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics /Chemistry Petrochemicals/Geology/Food Science and Quality Control /Computer Maintenance and Electronics)

(2013—2016 Admissions)

Time : Three Hours Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.
2. Define the average rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$.
3. State the intermediate value property of derivative of a function.
4. State the Mean Value theorem.
5. Give a function $f(x)$ whose derivative with respect to x is $\sin x$ and whose graph passes through the point (0, 2).
6. Define a monotonic function on an interval I.
7. If $f(x,y) = \frac{1}{\sqrt{x+y}}$ find $\frac{\partial f}{\partial x}$.
8. State the mixed derivative theorem for partial derivatives.
9. Define hyperbolic tangent function of x.
10. What is the period of $\cosh(x + iy)$.

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Find $\lim_{x \to 0} \frac{\sin 3x}{2x}$.
12. Find the equation to the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

Turn over

13. Show that the function $y = \sqrt{x}$ is not differentiable at $x = 0$.

14. Find the absolute minimum and maximum values of $f(x) = x^{2/3}$ on the interval $[-2, 3]$.

15. Show that the function $f(x) = x^4 + 3x + 1$ has exactly one real solution in $[-2, -1]$.

16. Find the value or values of C of the mean value theorem for the function $f(x) = x + \frac{1}{x}$ in $[1/2, 2]$.

17. If $w = x^2 + y^2, x = r - s, y = r + s$ then express $\frac{\partial w}{\partial r}$ in term of r and s.

18. If $f(x,y) = x\cos y + ye^x$ find $\frac{\partial^2 f}{\partial x \partial y}$ at $(\sqrt{2}, \pi/3)$.

19. Find f_{xyz} if $f(x,y,z) = 1 - 2xy^2z + x^2y$.

20. Find the imaginary part of $\tan(\alpha + \beta i)$.

21. Prove that $\cosh^2 y - \sinh^2 y = 1$.

22. Define $\sin x$ and $\cos x$ in terms of exponential functions and verify the result

$$\cos(x - y) = \cos x \cos y + \sin x \sin y.$$

$(8 \times 2 = 16)$

Part C (Short Essay Questions)

Answer any **six** *questions.*
Each question carries **4** *marks.*

23. By using the $\varepsilon - \delta$ definition of limit, show that $\lim_{x \to 1}\left(\frac{3}{2}x - 1\right) = \frac{1}{2}$.

24. State the sandwich theorem, using this find :

    (a) The horizontal asymptote of the curve $y = 2 + \frac{\sin x}{x}$.

    (b) Find $\lim_{\theta \to 0} \sin\theta$.

25. (a) Find the slope of the line, tangent to the curve $y = \sin^5 x$ at the point $x = \frac{\pi}{3}$.

    (b) Show that the slope of every line tangent to the curve $y = \frac{1}{(1-2x)^3}$ is positive.

26. State the first derivative test for the monotonic function and using this, find the critical point of $f(x) = x^3 - 12x - 5$, identify the intervals on which f is increasing and decreasing.

27. For what values of a, m and b does the function

$$f(x)=\begin{cases}3, & x=0\\ -x^2+3x+a, & 0<x<1\\ nx+b, & 1\le x\le 2\end{cases}$$

satisfying the hypothesis of the mean value theorem on the interval [0, 2].

28. Express $\frac{\sin 6\theta}{\sin\theta}$ in a series of descending powers of $\cos\theta$.

29. Sum to infinity the series $1 + c\cos\alpha + c^2\cos 2\alpha + \ldots$.

30. If resistors R_1, R_2 and R_3 ohms are connected in parallel to make a R-ohm resistor, the value of R can be found from $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30$, $R_2 = 45$, $R_3 = 90$ ohms.

31. Give a formula for implicit differentiation in terms of partial derivatives and use it to find $\frac{dy}{dx}$ if

(a) $xy+y^2-3x-3=0$; (b) $xe^y+\sin xy-\ln z=0$.

$(6\times 4 = 24)$

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) If $ax^2 + 2hxy + by^2 = 1$, where a, b, h are constants then show that

$$\frac{d^2y}{dx^2}=\frac{ab-h^2}{(ax+hy)^3}\left[hx^2+bxy+axy+hy^2\right].$$

(b) If $x=t+\frac{1}{t}$, $y=t-\frac{1}{t}$. Find $\frac{d^2y}{dx^2}$ as a function of t.

33. (a) Give a physical interpretation of mean value theorem. If $f'(x) = 0$ at every point of an open interval (a, b) then prove that $f(x)$ is constant for all x in (a, b).

(b) Find the critical points of $f(x)=x^{1/3}(x-4)$. Identify intervals on which f is increasing and decreasing. Find the functions local and absolute extreme values.

34. (a) Expand $\cos^5\theta \sin^7\theta$ in a series of multiples of θ.

(b) Prove that $\tanh(3x) = \dfrac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$.

(c) Prove that $\sinh(x+y)\cos(x-y) = \dfrac{1}{2}(\sinh 2x + \sinh 2y)$.

35. (a) Let $f(x,y) = \begin{cases} 0, & xy \neq 0 \text{ then} \\ 1, & xy = 0 \end{cases}$

(i) Find the limit of f as (x, y) approaches to $(0, 0)$ along the line $y = x$.

(ii) Show that both the partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ exist at the origin.

(b) Use the chain rule to find the derivative of $w = x^2y - y^2$ with respect to 't' along the path $x = \sin t$ and $y = e^t$. What is the derivative's value at $t = 0$.

$(2 \times 15 = 30)$